

# Cosmological wormholes

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We describe in details the procedure how the Lobachevsky space can be factorized to a space of the constant negative curvature filled with a gas of wormholes. We show that such wormholes have throat sections in the form of tori and are traversable and stable in the cosmological context. The relation of such wormholes to the dark matter phenomenon is briefly described. We also discuss the possibility of the existence of analogous factorizations for all types of homogeneous spaces.

## I. INTRODUCTION

It was demonstrated by us previously that all features of cold dark matter models (CDM) can be reproduced by the presence of a gas of wormholes [1, 2]. At very large scales wormholes behave like very heavy particles, while at smaller subgalactic scales they strongly interact with baryons and cure the problem of cusps. Lattice quantum gravity also suggests strong theoretical arguments for fractal properties of the topological structure of our Universe [3–5]. Such a structure has the most natural realization as a homogeneous gas of wormholes with fractal distribution of distances between wormhole entrances [6]. All such facts enforce us think that wormholes should play the key role in explaining the dark matter phenomenon.

However, there remains a strong scepticism in accepting the existence of actual wormholes. It bases, in the first place, on the fact that spherical wormholes are highly unstable (they collapse during the characteristic time  $\sim R/c$ , where  $R$  is the size of the throat section and  $c$  is the speed of light). Therefore, to be more or less stable (and traversable) they require the presence of an exotic violating the averaged null energy condition matter. It is possible to find a source of such matter at Planck scales (e.g., due to the Casimir effect) but such a matter does not exist at laboratory and astrophysical scales.

It turns out that the problem of the stability of cosmological wormholes can be easily solved when we consider less symmetric wormhole configurations [6]. In this case the presence of an exotic matter is not the necessary condition for a wormhole to be traversable in both directions and be stable, while the less symmetry gives rise to the fact that cosmological wormholes have the neck sections in the form of tori or even more complex surfaces. To avoid misunderstanding we point out that under a stable wormhole we do not mean static wormholes but rather wormholes whose the characteristic time of evolution and the rate of evolution are comparable with that of the Universe. In other words, such wormholes do survive till the present days, though they surely are not stationary

objects. In the present paper we explicitly demonstrate that a gas of wormholes can be obtained by a specific factorization of the Lobachevsky space, or, in a more general case, by the factorization of the Bianchi type - V homogeneous spaces [7]. Upon the factorization such spaces lose the homogeneity property. Nevertheless they remain to be locally homogeneous. This means that any sufficiently small portion of such spaces (i.e., any point with its neighborhood) coincides with analogous portion of the respective homogeneous space.

## II. OPEN COSMOLOGICAL MODEL

First, we recall basic properties of the open Friedman models. Consider the metric in the form

$$ds^2 = a^2(\tau) \left( d\tau^2 - \frac{4(d\vec{r})^2}{(1-r^2)^2} \right). \quad (1)$$

Here and below all symbols of vector operations (scalar product, etc) should be interpreted in purely formal way as if the coordinates  $\vec{r} = (x, y, z)$  were Cartesian. In particular,  $(d\vec{r})^2 = dx^2 + dy^2 + dz^2$  and  $r^2 = x^2 + y^2 + z^2$ . The scale factor  $a(\tau)$  is assumed to obey to the standard Friedman equations without any exotic matter.

The space-like part of the above metric represents the Lobachevsky space with the constant negative spatial curvature  $k = -1$ . In different coordinates it takes the form

$$dl^2 = \frac{4(d\vec{r})^2}{(1-r^2)^2} = d\chi^2 + sh^2\chi (d\theta^2 + \sin^2\theta d\phi^2). \quad (2)$$

In the coordinates  $\vec{r}$  the Lobachevsky space corresponds to the ball  $r^2 \leq 1$  while the boundary sphere  $r^2 = 1$  represents the absolute (i.e., the spatial infinity). This space possesses the properties of homogeneity and isotropy. The isotropy is straightforwardly seen from the above form of the metric (it has the invariant form under rotations  $x^{\alpha'} = U^\alpha_\beta x^\beta$ ,  $U \in O(3)$ ), while the homogeneity is not so explicit. It means that the metric keeps the invariant form with respect to translations  $x^{\alpha'} = T^\alpha(\vec{r}, \vec{r}_0)$

where the vector  $\vec{r}_0$  defines the value of the displacement. We point out that in the present and next sections the property of the isotropy of the space is not used and can be omitted.

Let  $\vec{r}_0$  be an arbitrary point of the ball  $r^2 \leq 1$  and define the translation which transforms this point to the origin  $\vec{r} = 0$ . To find the explicit law for the translation it is more convenient to use the Poincare model of the Lobachevsky space in the form of upper half-space. This is reached by introducing the Poincare variables (e.g., see [8, 9])

$$\vec{\eta} + \vec{c} = \frac{2(\vec{r} + \vec{c})}{(\vec{r} + \vec{c})^2}, \quad (3)$$

where  $\vec{c}$  is a point on the absolute. In terms of new coordinates the metric of the Lobachevsky space becomes

$$dl^2 = \frac{(d\vec{\eta})^2}{(\vec{\eta} \cdot \vec{c})^2} \quad (4)$$

and the ball  $r^2 \leq 1$  transforms into the half-space  $(\vec{\eta} \cdot \vec{c}) \geq 0$ , while the absolute becomes the plane  $(\vec{\eta} \cdot \vec{c}) = 0$ . Indeed, the relation (3) can be rewritten as

$$\vec{r} = \frac{2(\vec{\eta} + \vec{c})}{(\vec{\eta} + \vec{c})^2} - \vec{c}$$

and therefore

$$1 - r^2 = \frac{4(\vec{\eta} \cdot \vec{c})}{(\vec{\eta} + \vec{c})^2} \geq 0.$$

In terms of new coordinates it is easy to see the homogeneity of the Lobachevsky space with respect to translations, i.e., that the metric (4) does not change the form under the transformations

$$\vec{\eta}' = \lambda \vec{\eta} + \vec{\xi},$$

where  $\lambda > 0$  and  $\vec{\xi}$  is the constant vector orthogonal to  $\vec{c}$  (i.e.,  $(\vec{\xi} \cdot \vec{c}) = 0$ ). The vector  $\vec{\xi}$  describes translations along the directions orthogonal to  $\vec{c}$ , while the constant  $\lambda$  defines translations along the direction  $\vec{c}$ .

Let us take  $\vec{c} = \vec{r}_0/r_0$ . Then the new coordinates  $\vec{r}'$  (in which the point  $\vec{r}_0$  is at the origin) are

$$\vec{r}' = T_{r_0}(\vec{r}) = \frac{2(\vec{\eta}/\eta_0 + \vec{c})}{(\vec{\eta}/\eta_0 + \vec{c})^2} - \vec{c} \quad (5)$$

where  $\eta_0 = \frac{(1-r_0)}{(1+r_0)}$  and  $\vec{\eta}(r)$  is given by (3). This transformation describes simply the shift of space which reflects the homogeneity of space. In new coordinates the point  $\vec{r}_0$  transforms into the origin

$$\vec{r}'_0 = \vec{r}'(\vec{\eta}_0) = \vec{r}'(\vec{r}_0) = 0$$

while the metric remains the same  $dl^2 = \frac{4(d\vec{r}')^2}{(1-r'^2)^2}$ . We point out that translations have the group properties which is known to correspond to the type-V homogeneous space by the Bianchi classification [7].

### III. WORMHOLE CONNECTING TWO LOBACHEVSKY SPACES

The wormhole which connects two Lobachevsky spaces (or in a more general case two Bianchi type-V homogeneous spaces) is obtained from the Lobachevsky metric by a factorization, i.e., by imposing periodic boundary conditions with respect to some part of coordinates. Indeed, let us take an arbitrary point  $O$  in the space which we take as the origin of coordinates (this can be reached by the shift transformation  $T_{r_0}$  described in the previous section) and issue a geodesic line from it, which we take as the axis  $OX$ . Here we assume geodesics for the Lobachevsky metric (one should not mix them with spacetime geodesics which are trajectories of test particles in the total Friedman spacetime). Now consider two points on that line ( $x = \pm a$ ) which are at the geodesic distance  $d/2 = \ln \frac{1+a}{1-a} = -\ln \eta_a$  from the origin and issue from them a family of geodesics passing orthogonal to the axis  $OX$ . Such families of geodesics form the 2-dimensional hyper-surfaces which represent Lobachevsky planes and which are described by the equations

$$\pm \frac{(\vec{r} \cdot \vec{a})}{1+r^2} = \frac{a^2}{1+a^2}, \quad (6)$$

where  $\vec{a} = a\vec{\ell}$  and  $\vec{\ell} = (1, 0, 0)$  is the vector along  $OX$  (or the point at which the geodesic  $OX$  intersects the absolute). In a flat space such surfaces would remain to be parallel everywhere, while in the Lobachevsky space (of the constant negative curvature) they start to diverge and represent spheres which intersect the absolute under the right angles. Now let us identify (glue) points on these two surfaces and obtain a manifold which from the topological standpoint corresponds to a cylinder. Then the geodesic  $OX$  becomes the shortest closed geodesic on such a manifold (which has the length  $d$ ).

It may be convenient to describe such a gluing in terms of new coordinates which are introduced as follows (e.g., see appendix in [8])

$$\vec{u} = \frac{2\vec{r}'}{1+r'^2}. \quad (7)$$

In terms of new coordinates the absolute  $r^2 = 1$  does not change its position, i.e.  $u^2 = 1$  and the Lobachevsky space remains to be the ball  $u^2 \leq 1$ , while the above spheres become ordinary "flat" planes  $u_x a = (\vec{u} \cdot \vec{a}) = \pm \frac{2a^2}{1+a^2}$ .

Since the Lobachevsky space is homogeneous, it is possible to continue the cylinder to the whole Lobachevsky space. It is achieved by the translation of the fundamental region  $-\frac{2a}{(1+a^2)} \leq u_x \leq \frac{2a}{(1+a^2)}$  along the axis  $Ou$  on the distance  $d(a) = 2 \ln \frac{1+a}{1-a}$  (which is the shortest distance between the two surfaces). Then the Lobachevsky space splits into "stripes"  $u_n \leq u_x \leq u_{n+1}$ , where  $u_n = T_a^2(u_{n-1})$  is the translation (which is defined by the vector  $\vec{a}$ , see below) and  $n = 0, \pm 1, \pm 2, \dots (n \in \mathbb{Z})$  every

of each coincides with the fundamental region (i.e., represents the same fundamental region). In other words, the cylinder described above is the factorization of the Lobachevsky space over the translation  $u = T_a^2(u)$ , while the Lobachevsky space itself represents a cover of the fundamental region (of the cylinder).

The explicit law for such a translation is given by (5). Indeed, let us choose  $\vec{c} = \vec{\ell} = (1, 0, 0)$ , then in new coordinates the two planes ( $u_x = \pm \frac{2a}{(1+a^2)}$ ) become the two semi-spheres with the centers at the origin and radii

$$R_{\pm} = \left( \frac{1-a}{1+a} \right)^{\pm 1}.$$

The plane  $u_x = 0$  (or  $x = 0$ ) in such coordinates becomes the semi-sphere of the unit radius. Thus the translation along  $\vec{\ell}$  which transforms the spheres  $R_- \rightarrow R_+$  is given by  $\lambda$

$$\lambda = \frac{R_+}{R_-} = \left( \frac{1-a}{1+a} \right)^2$$

and the system of stripes on the ball  $u^2 \leq 1$  becomes the system of rings restricted by spheres of the radii

$$R_n = \lambda^n R_+.$$

This means that the cylinder is the factorization of the Lobachevsky space with respect to re-scaling with  $\lambda$ , i.e. any two points related by the relation  $\vec{\eta}' = \lambda^n \vec{\eta}$  correspond to the same point of the cylinder. In terms of initial coordinates the factorization looks like

$$\vec{r} \sim \vec{r}_n = T_a^{2n}(\vec{r}) = \frac{2(\lambda^n \vec{\eta} + \vec{\ell})}{(\lambda^n \vec{\eta} + \vec{\ell})^2} - \vec{\ell}, \quad (8)$$

where the sign  $\vec{r} \sim \vec{r}'$  means that these two points are equivalent (represent the same point) and

$$\vec{\eta} = \frac{2(\vec{r} + \vec{\ell})}{(\vec{r} + \vec{\ell})^2} - \vec{\ell}.$$

Thus, we see that the gluing of the two surfaces constructed does not change the metric at all. It however changes properties of the geodesic lines and matter fields defined on such a manifold. In particular, when we consider density perturbations  $\delta\rho(\vec{r}, t)$  on such a background it should obey the identity  $\delta\rho(\vec{r}, t) = \delta\rho(\vec{r}_n, t)$  since  $\vec{r}$  and  $\vec{r}_n$  represent the same point of the manifold. In all other respects the cylinder behaves like the standard Lobachevsky space.

It however should be noted that upon the factorization the space loses the property to be homogeneous. Indeed, the homogeneity means that all points are equivalent (any point can be taken as origin) and the space looks in the same way from any point of the space. This surely

does not retain upon the factorization. Indeed, consider the geodesic  $OX$  ( $-a \leq x \leq a$ ) which goes through the origin  $O$  and represents the shortest closed geodesic. If we take any other point  $P$  of the space which does not lie on  $OX$  axis and take it as origin, then we find that there are no closed geodesics at all which go through the point  $P$ . In other words such two points are not equivalent (the properties of space look in a different way from these two points). Instead of global homogeneity the factorized space possesses however the property of the local homogeneity. This means that a sufficiently small geodesic ball (in our case with geodesic distances less than  $d(a)$ ) around any point can be transformed (by an appropriate translation) into the analogous ball around any other point  $P$  (and it coincides with the analogous ball in the homogeneous Lobachevsky space).

Consider now an orthogonal to  $OX$  geodesic issued from the origin  $O$  which without loss of generality we take as the axis  $OY$  and construct in the same way an additional couple of hyper-surfaces which go through the points  $y = \pm b$  and are orthogonal to the axis  $OY$ . Now the shortest distance between such surfaces is  $d(b) = \ln\left(\frac{1+b}{1-b}\right)^2 = -\ln\lambda(b)$  and the surfaces are defined by the equations

$$\frac{2(\vec{r} \cdot \vec{b})}{1+r^2} = \pm \frac{2b^2}{1+b^2} = bu_y,$$

where  $\vec{b} = (0, b, 0)$ . Taking the point on the absolute  $\vec{c} = \vec{b}/b$  we define the analogous transformation (8) and an additional factorization of the Lobachevsky space with the equivalence  $\vec{r} \sim \vec{r}_m = T_b^{2m}(\vec{r})$  ( $m \in \mathbb{Z}$ ). The wormhole configuration is the manifold which is obtained from the Lobachevsky space by both factorizations, i.e., any two points are equivalent (the same), if they are related by

$$\vec{r} \sim \vec{r}_{n,m} = T_b^{2m}(T_a^{2n}(\vec{r}))$$

for any  $n, m \in \mathbb{Z}$ . The two vectors  $\vec{a}$  and  $\vec{b}$  are generators of such a factorization, while the point  $O$  defines the "central" point of the wormhole. We point out that the section  $z = 0$  is the homogeneous space (torus of the constant negative curvature) and all points on it can be taken as a "central" point of the wormhole.

#### IV. SECTIONS OF THE WORMHOLE

Now we are ready to describe the structure of sections of such a wormhole. The simplest picture appears in coordinates (7). For any  $u_z = \text{const}$  the section represents the torus, i.e., the rectangle

$$-\frac{2a}{1+a^2} \leq u_x \leq \frac{2a}{1+a^2}, \quad -\frac{2b}{1+b^2} \leq u_y \leq \frac{2b}{1+b^2} \quad (9)$$

with opposite sides identified. Whether these inequalities restrict a finite region or not depends on the behavior of

the absolute  $\bar{u}^2 = 1$ . On the  $u_x - u_y$  plane the absolute (i.e., the infinity) is given by  $u_x^2 + u_y^2 = 1 - u_z^2$ . If we assume  $a^2 + b^2 < 1$ , then for  $u_z = 0$  the torus (or the rectangle (9)) lies within the admissible domain  $u_x^2 + u_y^2 \leq 1$  and the section is the torus indeed. In particular, at  $u_z = 0$  such a torus has the smallest area. The section in the form of the torus retains with the increase of  $u_z^2$  till it reaches the value  $(u_z^0)^2 = 1 - \left(\frac{2a}{1+a^2}\right)^2 - \left(\frac{2b}{1+b^2}\right)^2$ . At this value four points  $(u_x = \pm \frac{2a}{1+a^2}, u_y = \pm \frac{2b}{1+b^2})$  reach the absolute which means that they lie at infinity and the torus starts to destroy. We point out that at  $u_z = \pm u_z^0$  the surface of the torus still has a finite area. Let  $a < b$ , then for  $u_z^2 > u_z^{*2} = 1 - \left(\frac{2a}{1+a^2}\right)^2$  the absolute  $(u_x^2 + u_y^2 = 1 - u_z^2)$  gets within the rectangle (9) which means that the inequalities (9) does not set any restriction on admissible values of  $u_x$  and  $u_y$  and, therefore, the section is not restricted. In other words, the sections  $u_z = \text{const}$  represent the sections of the unrestricted Lobachevsky space (without any factorization).

Thus, the two regions  $u_z > u_z^*$  and  $u_z < -u_z^*$  correspond to two parts of the standard Lobachevsky spaces which are connected only via the throat ( $u_z^2 < u_z^{*2}$ ) in the form of a torus. In other words such a manifold corresponds to a wormhole configuration which connects two Lobachevsky spaces (we recall that any section  $u_z = \text{const}$  divides the Lobachevsky space on two equal parts). If we identify these two spaces with the help of some of motions of the Lobachevsky space, we may obtain a wormhole which connects two different regions in the same Lobachevsky space which we describe below.

## V. HANDLE TYPE WORMHOLE

The handle type wormhole configuration is constructed from the above construction as follows. First, consider a point  $O_1$  with coordinates being  $\vec{r}_1$  and two generators  $\vec{a}_1$  and  $\vec{b}_1$  (for simplicity we assume  $(\vec{a}_1 \cdot \vec{b}_1) = 0$ , though this is not a necessary condition) which define the factorization of the Lobachevsky space to the wormhole configuration described in the previous section. In the explicit form the factorization is given by the equivalence relations  $\vec{r} \sim \vec{r}' = \mathbf{T}_A^{nm}(\vec{r}) = T_{r_1}^{-1} T_{a_1}^{2n} T_{b_1}^{2m} T_{r_1}(\vec{r})$ , where  $n, m \in \mathbb{Z}$ ,  $T_{r_1}$  defines the shift of the origin to the point  $O_1$ , and  $A_1$  defines the set of parameters  $A_1 = (\vec{r}_1, \vec{a}_1, \vec{b}_1)$ . Upon the factorization the Lobachevsky space reduces to the wormhole described in the previous section. The point  $O_1$  defines the central point of such a wormhole  $\vec{r}_1' = T_{r_1}(\vec{r}_1) = 0$ , while the regions  $z_1 > 0$  and  $z_1 < 0$  can be considered as two Lobachevsky spaces from which we cut a solid torus. Consider now an additional point  $O_2$  (let it be in the region  $z_1 > 0$  and lie outside the solid torus  $u_{z_1} > u_{z_1}^*$ ) and define another two generators  $\vec{a}_2$  and  $\vec{b}_2$  such that  $a_1 = |\vec{a}_1| = a_2$  and  $b_1 = b_2$ . Then they define another factorization and an additional wormhole

which also separates the upper part of the Lobachevsky space  $z_1 > 0$  into two parts  $z_2 > 0$  and  $z_2 < 0$ , which are also the two Lobachevsky spaces with a some region (in the form of solid torus) being removed. The relations  $a_1 = a_2$  and  $b_1 = b_2$  insure that such solid tori are equal and, therefore, we can match and identify the regions  $z_1 < 0$  and  $z_2 > 0$ . To match these two regions we have to use the composition of the translation from the point  $O_1$  to the point  $O_2$  (which is given by the map  $T_{1 \rightarrow 2} = T_{r_2}^{-1} T_{r_1}$ ) and the rotation  $U$  which matches the axis  $O_2 Z_2$  ( $z_2 > 0$ ) and  $-O_1 Z_1$  ( $z_1 < 0$ ). This assumes an additional factorization of the initial Lobachevsky space by the equivalence relation  $\vec{r} \sim \vec{r}' = U T_{r_2}^{-1} T_{r_1}(\vec{r})$ . The resulting manifold corresponds to the wormhole configuration which connects (through the handle in the form of a torus) two regions in the same space. From the visual standpoint such a space corresponds to a Lobachevsky space in which we cut two equal solid tori and glue them by the surfaces of the tori.

One particular degenerate configuration of such a wormhole serves of a special interest. It is the so-called string-like configuration. Indeed, it realizes in the situation when both points  $O_1$  and  $O_2$  belong to the same plane  $z_1 = z_2 = 0$ . In this case the additional factorization reduces to the reflection  $z_1 = -z_2$  and a shift on the two-dimensional plane  $z = 0$  ( $T_{1 \rightarrow 2}$ ). From visual standpoint in this case instead of two solid tori we will get a single torus (positions of the two tori merely coincide), while such a configuration corresponds to a closed cosmic string. In the case when  $b \rightarrow 1$  one of the torus radii becomes infinite and such a wormhole becomes an open cosmic string.

## VI. WORMHOLE IN A CONFORMALLY FLAT SPACETIME

It may be convenient to consider wormholes in a flat spacetime. Let us change the coordinates in the interval (1) according to

$$x^0 = e^\tau \frac{1+r^2}{1-r^2}, \quad \vec{x} = 2e^\tau \frac{\vec{r}}{1-r^2}$$

then the ordinary flat interval reduces to

$$ds^2 = (dx^0)^2 - (d\vec{x})^2 = e^{2\tau} \left( d\tau^2 - 4 \left( \frac{d\vec{r}}{1-r^2} \right)^2 \right)$$

therefore from (1) we find

$$ds^2 = a^2(\tau) e^{-2\tau} \left( (dx^0)^2 - (d\vec{x})^2 \right)$$

where the variable  $\tau$  is given by

$$e^{2\tau} = (x^0)^2 - (\vec{x})^2$$

and therefore, such coordinates do not cover the whole spacetime. In terms of new coordinates the hypersurfaces (6) become flat but are depending on time

$$(\vec{x} \cdot \vec{a}) = \pm \frac{2a^2}{1+a^2} x^0. \quad (10)$$

They move in space with the velocities

$$\vec{V} = \pm \frac{2\vec{a}}{1+a^2}.$$

Since  $a^2 \leq 1$ , such velocities are always less than the speed of light ( $V^2 \leq 1$ ) and can be very small (for  $a^2 \ll 1$ ,  $V \simeq 2a$ ). Thus in these coordinates the space-like part will represent the standard flat torus whose both radii expand with the speed proportional to their size. The bigger radius the bigger speed of expansion. We stress that such boundary conditions have the direct analogy to moving flat mirrors in the flat spacetime which were used in describing the general behavior of the metric near the cosmological singularity in classical and quantum gravity, e.g., see Refs. [10] and references therein.

## VII. DARK MATTER PHENOMENON

When the gravitational field is rather weak, the time component of the Einstein equations for scalar perturbations reduce to the standard Newton's equation [11]

$$\delta R_{00} \simeq \frac{1}{a^2} \Delta \phi = 4\pi G \left( \delta \rho + \frac{3}{c^2} \delta p \right), \quad (11)$$

here  $a$  is the scale factor of the Universe,  $R_{00}$  is the time component of the Ricci tensor,  $\phi$ ,  $\delta \rho$  and  $\delta p$  are the scalar metric, mass density and pressure perturbations respectively. The Laplace operator  $\Delta \phi = \frac{1}{\sqrt{\gamma}} \partial_\alpha (\sqrt{\gamma} \gamma^{\alpha\beta} \partial_\beta \phi)$  is constructed with the help of the metric (2). The presence of a factorization (of wormholes) changes properties of perturbations defined on such a manifold. In particular, the density perturbations  $\delta \rho(\vec{r}, t)$  should obey the identity  $\delta \rho(\vec{r}, t) = \delta \rho(\vec{r}', t)$  for any two points  $\vec{r} \sim \vec{r}' = T(\vec{r})$  related by one of possible equivalence relations (which means periodicity with respect to some appropriate coordinates). Therefore, the behavior of perturbations can be determined by the Green function which corresponds to a unit source

$$\Delta G(x, x') = \frac{4\pi}{\sqrt{\gamma}} \delta(r - r'). \quad (12)$$

First, let us set the position of the source  $r'$  to the origin (which is made by translation  $T_{r'}$  defined by (5)). Then the Green function (i.e., the Newton's law for the Lobachevsky space) looks like

$$G(r) = -\frac{1+r^2}{r}.$$

For  $r \ll 1$  it gives the standard Newton's law  $G \simeq -1/r$ . Now making use the back transformation  $T_{r'}^{-1}$  we find

$$G(r, r') = -\frac{1 + (T_{r'}^{-1}(\vec{r}))^2}{|T_{r'}^{-1}(\vec{r})|}.$$

The presence of a gas of wormhole means the existence of the factorization of space. This means that all points which are connected by transformations of the type  $r'_{N,k} = \mathbf{T}_{A_k}^{n_k m_k}(\vec{r}')$  represent the same point of space (where  $N$  numerates different wormholes and  $k$  all parameters  $A_k$ ,  $n_k$ , and  $m_k$  of a particular wormhole). Therefore, in terms of the Lobachevsky space (which represents a cover for the actual physical space) the unit source multiplies. It acquires an infinite number of images, while the right-hand side of (12) transforms into

$$\frac{1}{\sqrt{\gamma}} \delta(r - r') \rightarrow \frac{1}{\sqrt{\gamma}} \sum_{N,k} \delta(r - r'_{N,k}) = \frac{1}{\sqrt{\gamma}} \delta(r - r') + b(r, r').$$

This defines a halo of "dark matter" around any point source  $b(r, r')$ . The structure of such a halo is defined by the distribution of wormholes (i.e., by distributions of points  $O_k$  and generators  $\vec{b}_k, \vec{a}_k$ ). From the formal standpoint the equation for perturbations (11) remains correct (as the microscopic equation), but it requires very complex boundary conditions (on wormhole throats, or due to the factorization of the background space). Exactly like in macroscopic Electrodynamics it is more convenient to introduce the specific topological permeability and describe effects of wormholes (of the factorization) as a modification of the perturbation theory. In linear case, if we neglect possible peculiar motions of wormholes, it is given by the bias function [12] which transforms the right hand side of (11) according to

$$\delta \rho(r) = \delta \rho(r) + \int b(r, r') \delta \rho(r') d^3 r'. \quad (13)$$

In a more general case the bias function depends on time which more essentially modifies the perturbation theory. We do not discuss this problem here in more details, see however the more substantial discussions of this problem in Refs. [1, 2, 12].

## VIII. DISCUSSIONS

In the present paper we described in details the factorization of the Lobachevsky space to a constant negative curvature space which contains an arbitrary collection of wormholes. Such a space evolves as the open Friedmann model, while the presence of a gas of wormholes does not require any form of exotic matter. It is straightforward to generalize such a factorization on the case of a homogeneous Bianchi type- V homogeneous spaces, however, in this case the handle type wormhole will add some restrictions on possible orientations of the conjugated throat entrances.

There remains an important problem to study all possible factorizations of the rest Bianchi types of homogeneous spaces and verify which of them may correspond to non-trivial topological (wormhole-like or other) configurations. We point out that in general, upon the factorization the homogeneous space loses the property of global homogeneity, though the local homogeneity retains. The exclusion is the case when the factor group  $G/H$  possesses the group properties, i.e. represents a group. Here  $G$  is the group of homogeneity (group of translations) and  $H$  is a finite subgroup (e.g., the subgroup defined by a translation  $T_a$  on the distance  $a$  and all its degrees  $T_a^n$ ,  $n \in \mathbb{Z}$  which define a discrete subgroup of the group of translations). In this case the space remains to be globally homogeneous. The simplest example is the Bianchi type - I models. For this spaces translations is the standard shift on a constant vector as in the standard Euclidean space. The factorization with respect to a specific translation (displacement on a constant vector) transforms the Euclidean space in the ordinary cylinder (periodic in the direction of the above displacement). The cylinder also represents a homogeneous space, since in contrast to the wormhole in the Lobachevsky space, it does not contain distinguished points, while the factor group  $G/H$  represents a new group which acts on the cylinder. In this case the Bianchi type -I spaces represent the universal covering for the cylinder.

In this manner, we see that stable cosmological wormholes may indeed exist without any exotic matter and they may survive till the present days. A stable cosmological wormhole has the neck section in the form of a torus and, therefore, it may leave specific imprints in CMB in a form of rings. This may explain the results of Ref. [13] where the presence of such ring-type structures in CMB was established. See also discussion of this problem in Ref. [14].

In confronting to observations there remains a strong problem though. Indeed, observations of  $\Delta T/T$  spectrum indicate that the spatial curvature of our space is very close to zero, while the construction of a stable wormhole suggested seems to require the presence of the negative

curvature.

To overcome this problem we see two possible ways. First one is to introduce inhomogeneities. Indeed, the factorized Lobachevsky space is already an inhomogeneous space (though it may have the property of local homogeneity). In other words, every wormhole can be considered as an inhomogeneous structure in the background space. Therefore, in principle, we may add some amount of matter density outside of the wormhole necks to make the background space close to the flat space (the matter density should be closer to the critical value). However it is not possible to do for throats themselves which means that throats would expand a little bit faster than the background space. While the typical size of wormhole throats is much smaller, than the Hubble radius, this will not destroy the observed mean homogeneity and isotropy of the Universe. We point out that from the astrophysical standpoint wormhole throats look like specific compact astrophysical objects and the fact that they introduce some local inhomogeneity is not surprising. The second way is to study more complex wormhole-type configurations.

There also remains the problem to consider the modification of the perturbation theory in the presence of wormholes. From the microscopic (or formal) standpoint the Lifshitz theory [15] (see also [11]) does not change at all, since microscopic equations have local character and remain the same. However they require too complex boundary conditions (due to the factorization of the background space). The best way to account for the presence of wormholes is to introduce in the equations the permeability of space which have the topological origin. For example, the presence of wormholes may be described phenomenologically as the bias (13) which can be interpreted as the presence of the dark matter. This essentially simplifies the consideration of the behavior of perturbations. During the development of metric perturbations wormholes start to move and the total picture becomes very complex and even nonlinear. Some steps in this direction we did in Ref. [1] but those are clearly not sufficient.

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